

$$(3) \frac{1}{6} \arctan \frac{2}{3} x + C \quad (4) 2e^{\sqrt{x}} + C$$

$$4. (1) \frac{1}{2} x^2 - \frac{1}{2} \ln(1+x^2) + C$$

$$(2) \frac{1}{3} \ln|x-1| - \frac{1}{3} \ln|x+2| + C$$

$$(3) \frac{1}{4} \ln \left| \frac{2+x}{2-x} \right| + C$$

$$(4) \frac{\sqrt{2}}{4} \ln \left| \frac{\sqrt{2}x-1}{\sqrt{2}x+1} \right| + C$$

$$5. (1) \cos \frac{1}{x} + C$$

$$(2) \frac{1}{4a} \sin^4(ax+b) + C$$

$$(3) \frac{1}{2} x - \frac{1}{12} \sin(2-6x) + C$$

$$(4) \frac{3}{8} x - \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + C$$

$$6. (1) \frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C$$

$$(2) -\frac{1}{2} \ln|\cos 2t| - \frac{1}{2} \ln|\sec 2t + \tan 2t| + C - \frac{1}{2} \ln|t + b|$$

$$(3) -\frac{2}{3} (1 + \cot x)^{\frac{3}{2}} + C$$

$$(4) -\cot \frac{x}{2} + C$$

$$7. (1) -2 \sin^{-\frac{1}{2}} u + C$$

$$(2) \frac{1}{10} \sin 5x + \frac{1}{2} \sin x + C$$

$$(3) \frac{1}{4} \sin 2x - \frac{1}{16} \sin 8x + C$$

$$(4) x + C$$

$$\therefore (1) \arctan e^t + C \quad (2) -\sqrt{1-2 \ln x} + C$$

$$(3) e^{\arctan x} + C \quad (4) \ln \frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1} + C$$

$$9. (1) \frac{1}{2} \ln(x^2+2x+2) + \arctan(x+1) + C$$

$$(2) \arctan \frac{x}{3} - \frac{1}{2} \ln(9+x^2) + C$$

$$(3) \frac{\sqrt{2}}{2} \arctan \frac{x+1}{\sqrt{2}} + C$$

$$(4) \arcsin \frac{2x-1}{\sqrt{5}} + C$$

$$10. (1) 3 \left(\frac{\sqrt[3]{(x+1)^2}}{2} - \sqrt[3]{x+1} + \ln|1+\sqrt[3]{x+1}| \right) + C$$

$$(2) 2\sqrt{x-1} + 4\sqrt[4]{x-1} + 4 \ln|\sqrt[4]{x-1}-1| + C$$

$$(3) -\frac{2}{5} (\sqrt[5]{2-x})^5 + \frac{8}{3} (\sqrt[3]{2-x})^3 - 8\sqrt[2]{2-x} + C$$

$$(4) \sqrt{2x} - \ln(1+\sqrt{2x}) + C$$

$$11. (1) 2 \arcsin \frac{x}{2} - \frac{x\sqrt{4-x^2}}{2} + C \quad (2) \sqrt{1+x^2} + C$$

$$\arctan \frac{x}{a} - \frac{x}{2(x^2+a^2)} + C \quad (4) \frac{x}{\sqrt{1+x^2}} + C$$

$$12. (1) \sqrt{x^2-1} - \arccos \frac{1}{x} + C$$

$$(2) \frac{1}{2a^3} t + \frac{1}{4a^3} \sin 2t + C$$

$$(3) \ln|x+\sqrt{x^2-1}| + C$$

$$(4) \ln(x+\sqrt{x^2+2}) + C$$

$$13. (1) \frac{1}{a} \ln \left| \frac{\sqrt{x^2+a^2}-a}{x} \right| + C - \frac{1}{a^2} \frac{\sqrt{x^2+a^2}}{x} + C$$

$$(2) \ln \left| \frac{1}{\sqrt{1-x^2}} \right| + C$$

$$(3) -\frac{(1-x^2)^{\frac{3}{2}}}{3x^3} + C$$

$$(4) -\frac{(9-x^2)^{\frac{3}{2}}}{27x^3} + C - \frac{(3-x^2)^{\frac{3}{2}}}{9x^3} + C$$

习题 4-4

$$1. (1) \sin x - x \cos x + C \quad (2) x \tan x + \ln |\cos x| + C$$

$$(3) \frac{1}{8} \sin 2x - \frac{1}{4} x \cos 2x + C \quad (4) -\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} + C$$

$$2. (1) \frac{1}{2} x^2 \arcsin x - \frac{1}{4} \arcsin x + \frac{x}{4} \sqrt{1-x^2} + C$$

$$(2) \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$$

$$(3) \frac{x^3}{3} \ln x - \frac{1}{9} x^3 + C$$

$$(4) e^x f(x) + C$$

$$3. (1) x \arccos x - \sqrt{1-x^2} + C$$

$$(2) x \ln(1-x^2) - 2x + \ln \left| \frac{1+x}{1-x} \right| + C$$

$$(3) x(\ln x)^2 - 2x \ln x + 2x + C$$

$$4. (1) \frac{1}{2} (\cos x + \sin x) e^x + C$$

$$(2) \frac{1}{5} (\sin 2x - 2 \cos 2x) e^x + C$$

$$(3) -\frac{e^{-2x}}{3} (3 \cos 3x + 2 \sin 3x) + C$$

$$(4) \frac{e^{ax} (a \cos bx + b \sin bx)}{a^2 + b^2} + C$$

$$5. (1) (x+1) \operatorname{arccot} \sqrt{x} + \sqrt{x} + C$$

$$(2) 3e^{\sqrt[3]{x}} (\sqrt[3]{x^2} - 2\sqrt[3]{x} + 2) + C$$

$$(3) (\arcsin x)^2 + 2 \cos(\arcsin x) \cdot \arcsin x + C$$

$$(4) x - \arcsin x \cdot \cos(\arcsin x) + C$$

习题 4-5

$$1. (1) x^2 - 2x + 4 - \frac{8}{x+2}$$

$$(2) \frac{2}{3} \left(\frac{1}{x-1} + \frac{-21-x}{x^2+x+1} \right)$$

$$(3) \frac{1}{6(x+1)} + \frac{2}{15(x-2)} - \frac{3}{10(x+3)}$$

$$(4) \frac{1}{x} + \frac{2-x}{x^2+1}$$

$$\therefore (1) \frac{1}{3} x^3 - x^2 + 4x - 8 \ln|x+2| + C$$

$$(2) \ln|x-2| + C$$

$$(3) \frac{2}{3} \ln|x-1| - \frac{1}{3} \ln(x^2+x+1) - \frac{2\sqrt{3}}{3} \arctan \frac{2x+1}{\sqrt{3}} + C$$

$$(4) \frac{1}{6} \ln|x+1| + \frac{2}{15} \ln|x-2| - \frac{3}{10} \ln|x+3| + C$$

$$\therefore (1) \ln|x| - \frac{1}{2} \ln(1+x^2) + 2 \arctan x + C$$

$$(2) \frac{1}{2} \ln(1+x^2) - \frac{1}{2} \ln|x^2-x+1| + \frac{\sqrt{3}}{3} \arctan \frac{2x-1}{\sqrt{3}} + C$$

$$(3) \frac{1}{1+\sin x} - \ln \left| \frac{1+\sin x}{\sin x} \right| + C$$

$$(4) \frac{2\sqrt{3}}{3} \arctan \frac{\sqrt{3}}{3} \cdot \tan \frac{x}{2} + C$$

习题 4-6

$$(1) " \geq " \quad (2) " \leq " \quad (3) " \geq " \quad (4) " \leq "$$

$$2. (1) 4 \leq \int_1^3 (1+x) dx \leq 8$$

$$(2) \frac{\pi}{2} \leq \int_0^{\frac{\pi}{2}} (1 + \sin x) dx \leq \pi$$

$$(3) 6 \leq \int_1^4 (x^2 + 1) dx \leq 51$$

$$(4) \pi \leq \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (1 + \sin^2 x) dx \leq 2\pi$$

习题 4-7

$$1. (1) \sin x^2 \quad (2) -\frac{1}{\sqrt{1+x^2}}$$

$$(3) e^x \quad (4) \ln(1+x)$$

$$2. (1) -x^2 e^{-x} + 2x^3 e^{-x^2}$$

$$(2) \frac{1}{2\sqrt{x}} \sin x \quad (3) \frac{1}{2} \quad (4) 1$$

$$3. (1) \frac{1}{3} \quad (2) \frac{7}{12}\pi \quad (3) \ln 3 \quad (4) \frac{\pi}{6}$$

$$4. (1) \frac{5}{2} \quad (2) \frac{271}{6} \quad (3) 1 + \frac{\pi}{4} \quad (4) \sqrt{3} - \frac{\pi}{3}$$

$$5. (1) 1 - \frac{\pi}{4} \quad (2) 4 \quad (3) 0 \quad (4) \pi$$

习题 4-8

$$1. (1) \frac{\pi}{2}$$

$$(2) -\frac{4}{3} [(2\sqrt{2}-1)\sqrt{4-\sqrt{2}} - (2\sqrt{2}+1)\sqrt{4+\sqrt{2}}]$$

$$(3) 4 - 2\ln 3 \quad (4) \ln \frac{3}{2}$$

$$1. (1) \frac{\pi}{2} - \frac{4}{3} \quad (2) \frac{3}{2} \quad (3) 2 - \frac{\pi}{2} \quad (4) 3\ln 3$$

$$2. (1) \frac{1}{4} \quad (2) \frac{9\pi}{4} \quad (3) \frac{2}{3} \quad (4) 2\sqrt{2}$$

$$3. (1) 0 \quad (2) 0 \quad (3) \text{略} \quad (4) \text{略}$$

习题 4-9

$$1. (1) 1 - 2e^{-1} \quad (2) 8\ln 2 - 4 \quad (3) 1 \quad (4) \frac{1}{4}(e^2 + 1)$$

$$2. (1) \frac{1}{2}(1 + e \sin 1 - e \cos 1) \quad (2) \frac{\pi^3}{6} - \frac{\pi}{4}$$

$$(3) -\frac{2\pi}{\omega^2} \quad (4) \pi - 2$$

$$3. (1) \frac{\pi}{4} - \frac{1}{2} \quad (2) \frac{1}{2}(e^{\frac{\pi}{2}} - 1)$$

$$(3) \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1 \quad (4) \frac{\sqrt{3}}{3}\pi - \ln 2$$

$$4. (1) \pi^3 - 6\pi \quad (2) \ln \frac{\sqrt{6}}{2} - \frac{\sqrt{3}}{9}\pi + \frac{\pi}{4}$$

$$(3) 2 - \frac{3}{4\ln 2} \quad (4) \frac{1}{5}(e^{\pi} - 2)$$

习题 4-10

$$1. (1) 1 \quad (2) 1 \quad (3) \text{发散} \quad (4) \frac{1}{3}$$

$$2. (1) 1 - \frac{\pi}{4} \quad (2) \pi \quad (3) 0 \quad (4) \text{发散}$$

$$3. (1) 1 \quad (2) \frac{8}{3} \quad (3) \frac{\pi}{2} \quad (4) \frac{\pi}{2}$$

$$4. (1) -2 \quad (2) -\frac{1}{2}\ln 3 \quad (3) p \geq 1 \text{ 时发散. } p < 1 \text{ 时收敛于 } \frac{(b-a)^{1-p}}{1-p}$$

$$(4) \frac{3}{2}$$