

1. (1) $\{(x, y) | x < 1 \text{ 且 } y > 2\}$
 (2) $\{(x, y) | x^2 + y^2 \leq 1\}$

- (3) $\{(x, y) | 0 \leq x^2 + y^2 \leq 4\}$
 (4) $\{(x, y) | x + y > 0 \text{ 且 } x - y \geq 0\}$
 (5) $\{(x, y) | y \geq \sqrt{x} \text{ 且 } x \geq 0\}$
 (6) $\{(x, y) | -1 \leq x \leq 1 \text{ 且 } -1 \leq y \leq 1\}$
 (7) $\{(x, y) | x > y > 0\}$
 (8) $\{(x, y) | xy \geq 0\}$
 2. $x^2 + 3y^2$ 3. $\frac{4}{5}$, $-\frac{4}{5}$ 4. 证明略

习题 9-2

1. (1) 0 (2) 0 (3) $-\frac{1}{4}$ (4) 2 (5) $\ln 2$ (6) ∞
 2. 略
 3. 函数在 $(-\infty, 0) \cup (0, +\infty)$ 内连续.

习题 9-3

1. 10, 2
 2. (1) $\frac{\partial z}{\partial x} = 3x^2y + y^3$ $\frac{\partial z}{\partial y} = x^3 + 3xy^2$
 (2) $\frac{\partial z}{\partial x} = \frac{2x}{x^2 + y}$ $\frac{\partial z}{\partial y} = \frac{1}{x^2 + y}$
 (3) $\frac{\partial z}{\partial x} = \frac{2y}{(x+y)^2}$ $\frac{\partial z}{\partial y} = -\frac{2x}{(x+y)^2}$
 (4) $\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{x}} \sin \frac{x}{y} + \frac{\sqrt{x}}{y} \cos \frac{x}{y}$ $\frac{\partial z}{\partial y} = -\frac{x\sqrt{x}}{y^2} \cos \frac{x}{y}$
 (5) $\frac{\partial z}{\partial x} = \frac{2y}{(x+y)^2}$ $\frac{\partial z}{\partial y} = -\frac{2x}{(x+y)^2}$
 (6) $\frac{\partial z}{\partial x} = y^2(1+xy)^{y-1}$ $\frac{\partial z}{\partial y} = (1+xy)^y \left[\ln(1+xy) + \frac{xy}{1+xy} \right]$

- (7) $\frac{\partial z}{\partial x} = \frac{y}{x^2} \sin \frac{y}{x} + e^{xy}(1 + xy e^{xy})$
 $\frac{\partial z}{\partial y} = -\frac{1}{x} \sin \frac{y}{x} + x^2 e^{xy}$
 (8) $\frac{\partial z}{\partial x} = \frac{1}{2xy\sqrt{\ln(xy)}}$ $\frac{\partial z}{\partial y} = \frac{1}{2y\sqrt{\ln(xy)}}$
 4. (1) $\frac{\partial^2 z}{\partial x^2} = 2e^x \cos(x+y)$ $\frac{\partial^2 z}{\partial y^2} = -e^x \sin(x+y)$
 $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = e^x [\cos(x+y) - \sin(x+y)]$
 (2) $\frac{\partial^2 z}{\partial x^2} = 12x^2 - 8y^2$ $\frac{\partial^2 z}{\partial y^2} = 12y^2 - 8x^2$
 $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = -16xy$
 (3) $\frac{\partial^2 z}{\partial x^2} = \frac{-3x(x^2 + y^2)^{\frac{3}{2}} + 3x^3(x^2 + y^2)^{\frac{1}{2}}}{(x^2 + y^2)^3}$
 $\sqrt{\frac{\partial^2 z}{\partial y^2} = \frac{3xy^2(x^2 + y^2)^{\frac{1}{2}} - x(x^2 + y^2)^{\frac{3}{2}}}{(x^2 + y^2)^3}}$
 $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = \frac{3x^2y(x^2 + y^2)^{\frac{1}{2}} - y(x^2 + y^2)^{\frac{3}{2}}}{(x^2 + y^2)^3}$
 (4) $\frac{\partial^2 z}{\partial x^2} = y^x \ln^2 y$ $\frac{\partial^2 z}{\partial y^2} = x(x-1)y^{x-2}$
 $\frac{\partial^2 z}{\partial x \partial y} = xy^{x-1} \ln y + y^{x-1}$ $\frac{\partial^2 z}{\partial y \partial x} = y^{x-1} + x(x-1)y^{x-2}$
 (5) $\frac{\partial^2 z}{\partial x^2} = \frac{2xy}{(x^2 + y^2)^2}$ $\frac{\partial^2 z}{\partial y^2} = -\frac{2xy}{(x^2 + y^2)^2}$
 $\frac{\partial^2 z}{\partial x \partial y} = \frac{-x^2 + 3y^2}{(x^2 + y^2)^2}$ $\frac{\partial^2 z}{\partial y \partial x} = \frac{y^2 - x^2}{(x^2 + y^2)^2}$
 (6) $\frac{\partial^2 z}{\partial x^2} = y^2 e^{xy}$ $\frac{\partial^2 z}{\partial y^2} = x^2 e^{xy}$ $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = (1+xy)e^{xy}$
 5. 4, 6, 2

习题 9-4

- $dz = \frac{1}{3} dx + \frac{2}{3} dy$
- (1) $dz = \left(y - \frac{y}{x}\right) dx + \left(x + \frac{1}{x}\right) dy$
 (2) $dz = \frac{2x}{x^2 + y^2} dx + \frac{2y}{x^2 + y^2} dy$
 (3) $dz = \left(\frac{2x}{y^2} - \frac{2y^2}{x^3}\right) dx + \left(\frac{2y}{x^2} - \frac{2x^2}{y^3}\right) dy$
 (4) $dz = \frac{-xy}{(x^2 + y^2)^{\frac{3}{2}}} dx + \frac{x^2}{(x^2 + y^2)^{\frac{3}{2}}} dy$
 (5) $dz = [y \cos(xy) - y \sin(2xy)] dx + [x \cos(xy) - x \sin(2xy)] dy$
 (6) $dz = \frac{y}{x^2 + y^2} dx + \left(-\frac{x}{x^2 + y^2}\right) dy$
 (7) $du = yz x^{yz-1} dx + z x^{yz} \ln x dy + y x^{yz} \ln x dz$
 (8) $du = \frac{x}{\sqrt{x^2 + y^2 + z^2}} dx + \frac{y}{\sqrt{x^2 + y^2 + z^2}} dy + \frac{z}{\sqrt{x^2 + y^2 + z^2}} dz$
- (1) 2.975 (2) 0.98

习题 9-5

- $\frac{\partial z}{\partial x} = 6x^2 + 6y^2$ $\frac{\partial z}{\partial y} = 12xy$
- $\frac{\partial z}{\partial x} = \frac{2x(1+x)}{y^2} e^{2x+y}$ $\frac{\partial z}{\partial y} = -\frac{2x^2}{y^3} e^{2x+y} + \frac{x^2}{y^2} e^{2x+y}$
- $\frac{dz}{dt} = \frac{e^t + 6t^2}{e^t + 2t^3}$ 4. $\frac{dz}{dx} = \frac{2}{x^2} + e^{2x}(x + e^{2x})$
- $\frac{\partial z}{\partial x} = y(x^2 + y^2)^{xy} \left[\frac{2x^2}{x^2 + y^2} + y \ln(x^2 + y^2) \right]$

$$\frac{\partial z}{\partial y} = x(x^2 + y^2)^{xy} \left[\frac{2y^2}{x^2 + y^2} + x \ln(x^2 + y^2) \right]$$

$$6. \quad \frac{\partial^2 u}{\partial x^2} = f''_{11} + 2yf''_{12} + y^2 f''_{22} \quad \frac{\partial^2 u}{\partial y^2} = f''_{11} + 2xf''_{12} + x^2 f''_{22}$$

$$\frac{\partial^2 u}{\partial x \partial y} = f''_{11} + (x+y)f''_{12} + xyf''_{22} + f'_2$$

$$7. \quad \frac{\partial^2 u}{\partial x^2} = f''_{11} + 2yf''_{12} + y^2 f''_{22} \quad \frac{\partial^2 u}{\partial y^2} = y^3 f''_{22}$$

$$\frac{\partial^2 u}{\partial x \partial y} = xf''_{12} + xyf''_{22} + f'_2$$

习题 9-6

$$1. \quad \frac{\partial z}{\partial x} = \frac{yz - \sqrt{xyz}}{\sqrt{xyz} - xy} \quad \frac{\partial z}{\partial y} = \frac{xz - 2\sqrt{xyz}}{\sqrt{xyz} - xy}$$

$$2. \quad \frac{\partial z}{\partial x} = \frac{z}{x+z} \quad \frac{\partial z}{\partial y} = \frac{z^2}{y(x+z)}$$

$$3. \quad \frac{dy}{dx} = -\frac{y}{x}$$

$$4. \quad \frac{\partial^2 u}{\partial x^2} = \frac{x e^x - 2e^x + 2yz}{x^2 y} \quad \frac{\partial^2 z}{\partial y^2} = \frac{2z}{y^2} \quad \frac{\partial^2 z}{\partial x \partial y} = \frac{yz - e^x}{xy^2}$$

习题 9-7

- (1) 极小值为 -1 (2) 极大值为 8 (3) 极小值为 $-\frac{e}{2}$

$$(4) \text{ 极大值为 } \frac{500}{27} \quad 0$$

- (1) 最大值为 -2, 最小值为 -5

- (2) 最大值为 1, 最小值为 0

3. 长 $\sqrt[3]{2a}$, 宽 $\sqrt[3]{2a}$, 高 $\frac{1}{2}\sqrt[3]{2a}$

4. 长宽·高都为 $\frac{2}{3}a$