

共 36 分)

1. $xy' + y = e^x$ 满足 $y|_{x=1} = e$.

2. $(x+1)y' = -2xy + 2x$.

3. $xy'' - 2y' = x^3 + x$.

4. $y'' + 2y' + y = xe^x$.

5. $y'' - 2y' + 2y = \cos x$.

6. 求方程 $y' + \frac{2y}{y^2 - 6x} = 0$ 的通解.

7. $y'' - 5y' + 6y = e^{2x}$.

8. $(x^2 - 1)y' + 2xy - \cos x = 0$ 满足 $y(0) = 1$ 的特解.

9. $y'' + y' + 2 = 0$ 满足 $y(0) = 0, y'(0) = 0$ 的特解.

(四) (满分 10 分)

设 $f(x)$ 为连续函数, 且满足 $\int_0^x f(t) dt = e^x - f(x)$, 求 $f(x)$.

五、教材习题参考答案与自测题参考答案

习题 6-1

1. (1) 1 (2) 2 (3) 2 (4) 1 (5) 4 (6) 1

2. (1) 否 (2) 是 (3) 是 (4) 否

3. (1) $C=25$ (2) $C_1=0, C_2=1$

习题 6-2

1. (1) $\ln y = C|x|$ (2) $y = \frac{1}{5}x^3 + \frac{1}{2}x^2 + C$

(3) $\ln(1+e^x) + \ln|1-e^x| = C$ (4) $\sin x \cdot \sin y = C$

(5) $\frac{1}{3}(y+1)^3 + \frac{1}{4}x^4 = C$

2. (1) $2e^y - e^{2x} = 1$

(2) $|\sin x \cdot \ln y| = 1 - \cos x$

3. (1) $y = e^{-x}(x+C)$ (2) $y = (1+x)^2 \left(\frac{x^2}{2} + x + C \right)$

(3) $y = \frac{1}{x}(\pi - 1 - \cos x)$ (4) $y = \frac{8}{3} - \frac{2}{3e^{3x}}$

(5) $y = \frac{1}{2}x^3 + Cx$ (6) $y = e^{x^2}(\sin x + C)$

习题 6-3

1. (1) $y = x \arctan x - \frac{1}{2} \ln(1+x^2) + C_1x + C_2$

(2) $y = \frac{1}{12}x^4 + \cos x + C_1x^2 + C_2x + C_3$

(3) $y = -x + C_1e^x + C_2$

(4) $y = -\frac{1}{2}x^2e^{-x} - xe^{-x} - e^{-x} - C_1e^{-x} + C_2$

2. (1) $y = \frac{1}{6}x^3 \ln x - \frac{11}{36}x^3 + \frac{11}{36}$

(2) $y = -\frac{1}{a}e^{ax} + \frac{1}{a}$

习题 6-4

1. (1) $y = C_1e^{2x} + C_2e^{3x}$ (2) $y = C_1e^{-x} + C_2e^{\frac{x}{2}}$

(3) $y = e^{-2x}(C_1 + C_2x)$ (4) $y = e^{-3x}(C_1 \sin 2x + C_2 \cos 2x)$

2. (1) $y = C_1e^{3x} + C_2e^{-x} - \frac{e^x}{2} \left(\frac{1}{2}x + 1 \right)$

(2) $y = e^x(C_1 + C_2x) + \frac{1}{4}x^2e^x$

(3) $y = C_1e^{-x} + C_2e^{-2x} + \frac{1}{10}\cos x + \frac{3}{10}\sin x$

$$(4) y = \frac{1}{2}x \sin 2x + \frac{4}{3} \sin x + C_1 \cos 2x + C_2 \sin 2x$$

$$(5) y = C_1 \cos x + C_2 \sin x + \frac{1}{2}x \sin x + x^2 - 2$$

$$(6) y = e^{2x} \left(-\frac{1}{40} \cos 3x + \frac{3}{40} \sin 3x \right) + C_1 \cos x + C_2 \sin x$$

$$3. (1) y = 2x^2 - 7 + 7 \cos x - \sin x$$

$$(2) y = (x^2 - x + 1)e^x \oplus e^{-x}$$

自测题参考答案

(一) 选择题

题号	1	2	3	4	5	6	7	8	9	10
答案	B	C	B	C	C	C	D	C	D	C

(二) 填空题

$$1. y'' - y = 0$$

$$2. y'' - 2y' + y = 0$$

$$3. y'' = 0$$

$$4. y^2 = 2 \ln |x| - x^2 + C$$

$$5. y = 3 - \frac{3}{x}$$

$$6. x(Ax + B)$$

$$7. y = y_1 |x| + C e^{-\int p(x) dx}$$

$$8. s'' = t^{\frac{3}{2}}$$

(三) 解答题

$$1. y = \frac{1}{x} e^x$$

$$2. -\ln |1-y| = 2x - 2 \ln |x+1| + C$$

$$3. \text{设 } y' = u, \text{降阶可解得 } y' = x^3 - x + C_1 x^2, \text{从而 } y = \frac{1}{4} x^4 - \frac{1}{2} x^2 + \frac{C_1}{3} x^3 + C_2$$

$$4. y = (C_1 + C_2 x) e^{-x} + \frac{1}{4} (x-1) e^x$$

$$5. y = (C_1 \cos x + C_2 \sin x) e^x + \frac{1}{5} \cos x - \frac{2}{5} \sin x$$

$$6. x = y^3 \left(\frac{1}{2y} + C \right) \quad \text{提示: 将 } y \text{ 看作自变量, } x \text{ 是 } y \text{ 的函数.}$$

方程化为一阶线性微分方程 $\frac{dx}{dy} - \frac{3}{y}x = 0$.

$$7. y = C_1 e^{2x} + C_2 e^{3x} - x e^{2x}$$

$$8. y = \frac{\sin x - 1}{x^2 - 1}$$

$$9. y = 2 - 2x - 2e^{-x}$$

(四) 将方程两端分别对 x 求导, 可得 $f(x) = e^x - f'(x)$, 认为 $y = f(x)$, 上述方程为一阶线性微分方程 $y' + y = e^x$ 且满足

$$y(0) = 1. \text{解得 } f(x) = \frac{1}{2}(e^x - e^{-x}).$$